169. Time & Space

In the condition of God-self-realization, space, time and infinity *do not* exist.

Time, space and infinity are concepts of the ego-I condition.

All time exists simultaneously. Therefore, **all** events are fixed and knowable in advance as well as during and after the fact of their apparent "happening."

Even so, the knowledge of any event depends on our ability to enter into the plane or moment of that event. Therefore, knowledge of events outside of conventional memory and perception depends on our ability to transcend the body-mind in its present space-time state, configuration, or definition. And true knowledge of what is not contained in the present space-time limits of our experience depends on self-surrender, deep consciousness, ecstasy or self-transcendence, and resort to ignorance, or the Condition of Being that transcends all past and present knowledge. In fact, then, the same requirements exist as a condition of perfect memory, foreknowledge, and total knowledge that exist as the Ultimate Condition of Transcendental Ecstasy or God-Realization. Such is the Paradox or Equation of Reality. The same Condition pertains at Zero, Everything, and Anything.

SPACE-TIME by Albert Einstein

All our thoughts and concepts are called up by sense-experiences and have a meaning only in reference to these sense-experiences. On the other hand, however, they are products of the spontaneous activity of our minds; they are thus in no wise logical consequences of the contents of these sense-experiences. If, therefore, we wish to grasp the essence of a complex of abstract notions we must for the one part investigate the mutual relationships between the concepts and the assertions made about them; for the other, we must investigate how they are related to the experiences.

So far as the way is concerned in which concepts are connected with one another and with the experiences there is no difference of principle between the concept-systems of science and those of daily life. The concept-systems of science have grown out of those of daily life and have been modified and completed according to the objects and purposes of the science in question.

The more universal a concept is the more frequently it enters into our thinking; and the more indirect its relation to sense-experience, the more difficult it is for us to comprehend its meaning; this is particularly the case with pre-scientific concepts that we have been accustomed to use since childhood. Consider the concepts referred to in the words "where," "when," "why," "being," to the elucidation of which innumerable volumes of philosophy have been devoted. We fare no better in our speculations than a fish which should strive to become clear as to what is water.

SPACE

In the present article we are concerned with the meaning of "where," that is, of space. It appears that there is no quality contained in our individual primitive sense-experiences that may be designated as spatial. Rather, what is spatial appears to be a sort of order of the material objects of experience. The concept "material object" must therefore be available if concepts concerning space are to be possible. It is the logically primary concept. This is easily seen if we analyse the spatial concepts for example, "next to," "touch," and so forth, that is, if we strive to become aware of their equivalents in experience. The concept "object" is a means of taking into account the persistence in time or the continuity, respectively, of certain groups of experience-complexes. The existence of objects is thus of a conceptual nature, and the meaning of the concepts of objects depends wholly on their being connected (intuitively) with groups of elementary sense-experiences. This connection is the basis of the illusion which makes primitive experience appear to inform us directly about the relation of material bodies (which exist, after all, only in so far as they are thought).

In the sense thus indicated we have (the indirect) experience of the contact of two bodies. We need do no more than call attention to this, as we gain nothing for our present purpose by singling out the individual experiences to which this assertion alludes. Many bodies can be brought into permanent contact with one another in manifold ways. We speak in this sense of the position-relationships of bodies (*Lagenbeziehungen*). The general laws of such position-relationships are essentially the concern of geometry. This holds, at least, if we do not wish to restrict ourselves to regarding the propositions that occur in this branch of knowledge merely as relationships between empty words that have been set up according to certain principles.

Pre-scientific Thought.--Now, what is the meaning of the concept "space" which we also encounter in pre-scientific thought? The concept of space in pre-scientific thought is characterised by the sentence: "we can think away things but not the space which they occupy." It is as if, without having had experience of any sort, we had a concept, nay even a presentation, of space and as if we ordered our sense-experiences with the help of this concept, present *a priori*. On the other hand, space appears as a physical reality, as a thing which exists independently of our thought, like material objects. Under the influence of this view of space the fundamental concepts of geometry: the point, the straight line, the plane, were even regarded as having a self-evident character. The fundamental principles that deal with these configurations were regarded as being necessarily valid and as having at the same time an objective content. No scruples were felt about ascribing an objective meaning to such statements as "three empirically given bodies (practically infinitely small) lie on one straight line," without demanding a physical definition for such an assertion. This blind faith in evidence and in the immediately real meaning of the concepts and propositions of geometry became uncertain only after non-Euclidean geometry had been introduced.

Reference to the Earth.--If we start from the view that all spatial concepts are related to contact-experiences of solid bodies, it is easy to understand how the concept "space" originated, namely, how a thing independent of bodies and yet embodying their position-possibilities (*Lagerungsmöglichkeiten*) was posited. If we have a system of bodies in contact and at rest relatively to one another, some can be replaced by others. This property of allowing substitution is interpreted as "available space." Space denotes the property in virtue of which rigid bodies can occupy different positions. The view that space is something with a unity of its own is perhaps due to the circumstance that in pre-scientific thought all positions of bodies were referred to one body (reference body), namely the earth. In scientific thought the earth is represented by the co-ordinate system. The assertion that it would be possible to place an unlimited number of bodies next to one another denotes that space is infinite. In pre-scientific thought the concepts "space" and "time" and "body of reference" are scarcely differentiated at all. A place or point in space is always taken to mean a material point on a body of reference.

Euclidean Geometry.--If we consider Euclidean geometry we clearly discern that it refers to the laws regulating the positions of rigid bodies. It turns to account the ingenious thought of tracing back all relations concerning bodies and their relative positions to the very simple concept "distance" (*Strecke*). Distance denotes a rigid body on which two material points (marks) have been specified. The concept of the equality of distances (and angles) refers to experiments involving coincidences; the same remarks apply to the theorems on congruence. Now, Euclidean geometry, in the form in which it has been handed down to us from Euclid, uses the fundamental concepts "straight line" and "plane" which do not appear to correspond, or at any rate, not so directly, with experiences concerning the position of rigid bodies. On this it must be remarked that the concept of the straight line may be reduced to that of the distance.* Moreover, geometricians were less concerned with bringing out the relation of their fundamental concepts to experience than with deducing logically the geometrical propositions from a few axioms enunciated at the outset.

Let us outline briefly how perhaps the basis of Euclidean geometry may be gained from the concept of distance.

We start from the equality of distances (axiom of the equality of distances). Suppose that of two unequal distances one is always greater than the other. The same axioms are to hold for the inequality of distances as hold for the inequality of numbers.

*A hint of this is contained in the theorem: "the straight line is the shortest connection between two points." This theorem served well as a definition of the straight line, although the definition played no part in the logical texture of the deductions.

Three distances AB1, BC1, CA1 may, if CA1 be suitably chosen, have their marks BB1, CC1, AA1 superposed on one another in such a way that a triangle ABC results. The distance CA1 has an upper limit for which this construction is still just possible. The points A, (BB') and C then lie in a "straight line" (definition). This leads to the concepts: producing a distance by an amount equal to itself; dividing a distance into equal parts; expressing a distance in terms of a number by means of a measuring-rod (definition of the space-interval between two points).

When the concept of the interval between two points or the length of a distance has been gained in this way we require only the following axiom (Pythagoras' theorem) in order to arrive at Euclidean geometry analytically.

To every point of space (body of reference) three numbers (co-ordinates) x, y, z may be assigned--and conversely--in such a way that for each pair of points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) the theorem holds:

measure-number

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

All further concepts and propositions of Euclidean geometry can then be built up purely logically on this basis, in particular also the propositions about the straight line and the plane.

These remarks are not, of course, intended to replace the strictly axiomatic construction of Euclidean geometry. We merely wish to indicate plausibly how all conceptions of geometry may be traced back to that of distance. We might equally well have epitomised the whole basis of Euclidean geometry in the last theorem above. The relation to the foundations of experience would then be finished by means of a supplementary theorem.

The co-ordinate may and *must* be chosen so that two pairs of points separated by equal intervals, as calculated by the help of Pythagoras' theorem, may be made to coincide with one and the same suitably chosen distance (on a solid).

The concepts and propositions of Euclidean geometry may be derived from Pythagoras' proposition without the introduction of rigid bodies; but these concepts and propositions would not then have contents that could be tested. They are not "true" propositions but only logically correct propositions of purely formal content.

Difficulties.--A serious difficulty is encountered in the above represented interpretation of geometry in that the rigid body of experience does not correspond *exactly* with the geometrical body. In stating this I am thinking less of the fact that there are no absolutely definite marks than that temperature, pressure and other circumstances modify the laws relating to position. It is also to be recollected that the structural constituents of matter (such as atom and electron, *q. v.*) assumed by physics are not in principle commensurate with rigid bodies, but that nevertheless the concepts of geometry are applied to them and to their parts. For this reason consistent thinkers have been disinclined to allow real contents of facts (*reale Tatsachenbestände*) to correspond to geometry alone. They considered it preferable to allow the content of experience (*Erfahrungsbestände*) to correspond to geometry and physics conjointly.

This view is certainly less open to attack than the one represented above; as opposed to the atomic theory it is the only one that can be consistently carried through. Nevertheless, in the opinion of the author it would not be advisable to give up the first view, from which geometry derives its origin. This connection is essentially founded on the belief that the ideal rigid body is an abstraction that is well rooted in the laws of nature.

Foundations of Geometry.--We come now to the question: what is *a priori* certain or necessary, respectively in geometry (doctrine of space) or its foundations? Formerly we thought everything--yes, everything; nowadays we think--nothing. Already the distance-concept is logically arbitrary; there need be no things that correspond to it, even approximately. Something similar may be said of the concepts straight line, plane, of three-dimensionality and of the validity of Pythagoras' theorem. Nay, even the continuum-doctrine is in no wise given with the nature of human thought, so that from the epistemological point of view no greater authority attaches to the purely topological relations than to the others.

I>Earlier Physical Concepts.--We have yet to deal with those modifications in the space-concept, which have accompanied the advent of the theory of relativity. For this purpose we must consider the space-concept of the earlier physics from a point of view different from that above. If we apply the theorem of Pythagoras to infinitely near points, it reads

$$ds2 = dx2 + dy2 + dz2$$

where ds denotes the measurable interval between them. For an empirically-given ds the co-ordinate system is not yet fully determined for every combination of points by this equation. Besides being translated, a co-ordinate system may also be rotated.*This signifies analytically: the relations of Euclidean geometry are covariant with respect to linear orthogonal transformations of the co-ordinates.

In applying Euclidean geometry to pre-relativistic mechanics a further indeterminateness enters through the choice of the co-ordinate system: the state of motion of the co-ordinate system is arbitrary to a certain degree, namely, in that substitutions of the co-ordinates of the form

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

also appear possible. On the other hand, earlier mechanics did not allow co-ordinate systems to be applied of which the states of motion were different from those expressed in these equations. In this sense we speak of "inertial systems." In these favoured-inertial systems we are confronted with a new property of space so far as geometrical relations are concerned. Regarded more accurately, this is not a property of space alone but of the four-dimensional continuum consisting of time and space conjointly.

Appearance of Time.--At this point time enters explicitly into our discussion for the first time. In their applications space (place) and time always occur together. Every event that happens in the world is determined by the space-co-ordinates x, y, z, and the time-co-ordinate t. Thus the physical description was four-dimensional right from the beginning. But this four-dimensional continuum seemed to resolve itself into the three-dimensional continuum of space and the one-dimensional continuum of time. This apparent resolution owed its origin to the illusion that the meaning of the concept "simultaneity" is self-evident, and this illusion arises from the fact that we receive news of near events almost instantaneously owing to the agency of light.

This faith in the absolute significance of simultaneity was destroyed by the law regulating the propagation of light in empty space or, respectively, by the Maxwell-Lorentz electrodynamics. Two infinitely near points can be connected by means of a light-signal if the relation

*Change of direction of the co-ordinate axes while their orthogonality is preserved.

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = 0$$

holds for them. It further follows that ds has a value which, for arbitrarily chosen infinitely near space-time points, is independent of the particular inertial system selected. In agreement with this we find that for passing from one inertial system to another, linear equations of transformation hold which do not in general leave the time-values of the events unchanged. It thus became manifest that the four-dimensional continuum of space cannot be split up into a time-continuum and a space-continuum except in an arbitrary way. This invariant quantity ds may be measured by means of measuring-rods and clocks.

Four-Dimensional Geometry.--On the invariant ds a four-dimensional geometry may be built up which is in a large measure analogous to Euclidean geometry in three dimensions. In this way physics becomes a sort of statics in a four-dimensional continuum. Apart from the difference in the number of dimensions the latter continuum is distinguished from that of Euclidean geometry in that ds^2 may be greater or less than zero. Corresponding to this we differentiate between time-like and space-like line-elements. The boundary between them is marked out by the element of the "light-cone" $ds^2 = 0$ which starts out from every point. If we consider only elements which belong to the same time-value, we have

$$-ds^2 = dx^2 + dy^2 + dz^2$$

These elements ds may have real counterparts in distances at rest and, as before, Euclidean geometry holds for these elements.

Effect of Relativity, Special and General.--This is the modification which the doctrine of space and time has undergone through the restricted theory of relativity. The doctrine of space has been still further modified by the general theory of relativity, because this theory denies that the three-dimensional spatial section of the space-time continuum is Euclidean in character. Therefore it asserts that Euclidean geometry does not hold for the relative positions of bodies that are continuously in contact.

For the empirical law of the equality of inertial and gravitational mass led us to interpret the state of the continuum, in so far as it manifests itself with reference to a non-inertial system, as a gravitational field and to treat non-inertial systems as equivalent to inertial systems. Referred to such a system, which is connected with the inertial system by a non-linear transformation of the co-ordinates, the metrical invariant ds^2 assumes the general form:--

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu$$

where the $g_{\mu\nu}$'s are functions of the co-ordinates and where the sum is to be taken over the indices for all combinations 11, 12, . . .44. The variability of the $g_{\mu\nu}$'s is equivalent to the existence of a gravitational field. If the gravitational field is sufficiently general it is not possible at all to find an inertial system, that is, a co-ordinate system with reference to which ds^2 may be expressed in the simple form given above:--

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

But in this case, too, there is in the infinitesimal neighbourhood of a space-time point a local system of reference for which the last-mentioned simple form for ds holds.

This state of the facts leads to a type of geometry which Riemann's genius created more than half a century before the advent of the general theory of relativity of which Riemann divined the high importance for physics.

Riemann's Geometry.--Riemann's geometry of an n-dimensional space bears the same relation to Euclidean geometry of an n-dimensional space as the general geometry of curved surfaces bears to the geometry of the plane. For the infinitesimal neighbourhood of a point on a curved surface there is a local co-ordinate system in which the distance ds between two infinitely near points is given by the equation

$$ds^2 = dx^2 + dy^2$$

For any arbitrary (Gaussian) co-ordinate-system, however, an expression of the form

$$ds^2 = g_{11}dx^2 + 2g_{12}dx_1dx_2 + g_{22}dx_2^2$$

holds in a finite region of the curved surface. If the $g_{\mu\nu}$'s are given as functions of x_1 and x_2 the surface is then fully determined geometrically. For from this formula we can calculate for every combination of two infinitely near points on the surface the length ds of the minute rod connecting them; and with the help of this formula all networks that can be constructed on the surface with these little rods can be calculated. In particular, the "curvature" at every point of the surface can be calculated; this is the quantity that expresses to what extent and in what way the laws regulating the positions of the minute rods in the immediate vicinity of the point under consideration deviate from those of the geometry of the plane.

This theory of surfaces by Gauss has been extended by Riemann to continua of any arbitrary number of dimensions and has thus paved the way for the general theory of relativity. For it was shown above that corresponding to two infinitely near space-time points there is a number ds which can be obtained by measurement with rigid measuring-rods and clocks (in the case of time-like elements, indeed, with a clock alone). This quantity occurs in the mathematical theory in place of the length of the minute rods in three-dimensional geometry. The curves for which • ds has stationary values determine the paths of material points and rays of light in the gravitational field, and the "curvature" of space is dependent on the matter distributed over space.

Just as in Euclidean geometry the space-concept refers to the position-possibilities of rigid bodies, so in the general theory of relativity the space-time-concept refers to the behaviour of rigid bodies and clocks. But the space-time-continuum differs from the space-continuum in that the laws regulating the behaviour of these objects (clocks and measuring-rods) depend on where they happen to be. The continuum (or the quantities that describe it) enters explicitly into the laws of nature, and conversely these properties of the continuum are determined by physical factors. The relations that connect space and time can no longer be kept distinct from physics proper.

Nothing certain is known of what the properties of the space-time-continuum may be as a whole. Through the general theory of relativity, however, the view that the continuum is infinite in its time-like extent but finite in its space-like extent has gained in probability.

The physical time-concept answers to the time-concept of the extra-scientific mind. Now, the latter has its root in the time-order of the experiences of the individual, and this order we must accept as something primarily given.

I experience the moment "now," or, expressed more accurately, the present sense-experience (*Sinnen-Erlebnis*) combined with the recollection of (earlier) sense-experiences. That is why the sense-experiences seem to form a series, namely the time-series indicated by "earlier" and "later." The experience-series is thought of as a one-dimensional continuum. Experience-series can repeat themselves and can then be recognised. They can also be repeated inexactly, wherein some events are replaced by others without the character of the repetition becoming lost for us. In this way we form the time-concept as a one-dimensional frame which can be filled in by experiences in various ways. The same series of experiences answer to the same subjective time-intervals.

The transition from this "subjective" time (*Ich-Zeit*) to the time-concept of pre-scientific thought is connected with the formation of the idea that there is a real external world independent of the subject. In this sense the (objective) event is made to correspond with the subjective experience. In the same sense there is attributed to the "subjective" time of the experience a "time" of the corresponding "objective" event. In contrast with experiences external events and their order in time claim validity for all subjects.

This process of objectification would encounter no difficulties were the time-order of the experiences corresponding to a series of external events the same for all individuals. In the case of the immediate visual perceptions of our daily lives, this correspondence is exact. That is why the idea that there is an objective time-order became established to an extraordinary extent. In working out the idea of an objective world of external events in greater detail, it was found necessary to make events and experiences depend on each other in a more complicated way. This was at first done by means of rules and modes of thought instinctively gained, in which the conception of space plays a particularly prominent part. This process of refinement leads ultimately to natural science.

The measurement of time is effected by means of clocks. A clock is a thing which automatically passes in succession through a (practically) equal series of events (period). The number of periods (clock-time) elapsed serves as a measure of time. The meaning of this definition is at once clear if the event occurs in the immediate vicinity of the clock in space; for all observers then observe the same clock-time simultaneously with the event (by means of the eye) independently of their position. Until the theory of relativity was propounded it was assumed that the conception of simultaneity had an absolute objective meaning also for events separated in space.

This assumption was demolished by the discovery of the law of propagation of light. For if the velocity of light in empty space is to be a quantity that is independent of the choice (or, respectively, of the state of motion) of the inertial system to which it is referred, no absolute meaning can be assigned to the conception of the simultaneity of events that occur at points separated by a distance in space. Rather, a special time must be allocated to every inertial system. If no co-ordinate system (inertial system) is used as a basis of reference there is no sense in asserting that events at different points in space occur simultaneously. It is in consequence of this that space and time are welded together into a uniform four-dimensional continuum.

-A. Einstein